## Background

Length selectivities in an age model need to have the numbers-at-age converted into numbers-at-length via the age-length growth relationship which includes the spread of lengths around the mean length for a particular age.

The problem is that the selectivity and length distribution-at-age are convoluted over each length interval and so cannot be integrated analytically or approximated accurately. Hence, we must numerically integrate their product over each length class intervals, as in

, where *L(x)* could be the logistic ogive, and *s(x)* is the probability density function of fish sizes.

This must be done for each length interval. The numerical method we use is the extended trapezoidal rule using equally spaced points:

\int_a^b f(x)\,dx \approx \frac{b-a}{n} \left( {f(a) \over 2} + \sum_{k=1}^{n-1} \left( f \left( a+k \frac{b-a}{n} \right) \right) + {f(b) \over 2} \right)

where f(x) is the product L(x) s(x) and *a* is the lower bound of the length class and *b* is the upper bound (up to, but not including the lower bound for the next length class).

## Specific commands that are extra to observation-at-length

integrate\_method <type>

Comment: Method used to determine proportions in each length class

Type: <string>, one of “CASAL”,”SS3” [Leave a stub for “SS3”]

Default: CASAL

Value: CASAL & SS3 are numerical integration of the normal or lognormal curves; these also need *n\_integrate* to be specified.

n\_integrate <type>

Comment: Number of intervals or step size for integrating the normal curve

Type: integer

Default: 10

Value: Should be greater than 0. If *integrate\_method* = CASAL, then *n\_integrate* specifies the number of intervals within each length class for its integration. If *integrate\_method* = SS3, then 1/*n\_integrate* specifies the interval size in units of length, .e.g., 10 means intervals of 1 mm for cm as the length unit.

CASAL, proportions by length class

Each length class, assign *n\_integrate* + 1 equally spaced points across the length class starting with *class\_min[i]* and finishing at *class\_min[i + 1]*. Evaluate the normal curve at these points; call it the *NORM\_i* vector.

NORM\_i[j] = exp(-0.5 \* power( (length\_j – mean\_length)/standard\_deviation,2) / (sqrt(2 \* standard\_deviation)

The integral is given by {( *NORM\_i*[1] +  *NORM\_i*[*n\_integrate* + 1])/2 + sum\_j\_from\_2\_to\_ *n\_integrate* (*NORM\_i*[j])} times the spacing distance. To make sure they add up to one, the column should be divided by the sum of the column.

SS3, proportions by length class LEAVE A STUB, implement later

Here, the points within a length class are set to a fixed value. Now *n\_integrate* gives the number intervals in one length unit so the spacing distance is 1/ *n\_integrate*. Within each length class, get points at the spacing distance to cover the class. Potentially, each class could have a different number of points. Again, evaluate the normal curve and apply an analogous integration as given above.